

Large volume supersymmetry breaking without decompactification problem

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Abstract We consider heterotic string backgrounds in four-dimensional Minkowski space, where $\mathcal{N} = 1$ supersymmetry is spontaneously broken at a low scale $m_{3/2}$ by a stringy Scherk-Schwarz mechanism. We review how the effective gauge couplings at 1-loop may evade the “decompactification problem”, namely the proportionality of the gauge threshold corrections, with the large volume of the compact space involved in the supersymmetry breaking.

1 Introduction

A sensible physical theory must at least meet two requirements : Be realistic and analytically under control. The first point can be satisfied by considering string theory, which has the advantage to be, at present time, the only setup in which both gravitational and gauge interactions can be described consistently at the quantum level. In this review, we do not consider cosmological issues and thus analyze models defined classically in four-dimensional Minkowski space. The “no-scale models” are particularly interesting since, by definition, they describe in supergravity or string theory classical backgrounds, in which supersymmetry is spontaneously broken at an arbitrary scale $m_{3/2}$ in flat space [1]. In other words, even if supersymmetry is not explicit, the classical vacuum energy vanishes.

The most conservative way to preserve analytical control is to ensure the validity of perturbation theory. In string theory, quantum loops can be evaluated explicitly, when the underlying two-dimensional conformal field theory is itself under control. Clearly, this is the case, when one considers free field on the world sheet, for instance in toroidal orbifold models [2] or fermionic constructions [3]. In these frameworks, the $\mathcal{N} = 1 \rightarrow \mathcal{N} = 0$ spontaneous breaking of supersymmetry can be implemented at tree level *via* a stringy version [4] of the Scherk-Schwarz mechanism [5].¹ In this case, the supersymmetry breaking scale is of order of the inverse

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¹ Note that non-perturbative mechanisms based on gaugino condensation could also be considered, but only at the level of the low energy effective supergravity, thus at the price of loosing part of the string predictability.

volume of the internal directions involved in the breaking. For a single circle of radius R , one has

$$m_{3/2} = \frac{M_s}{R}, \quad (1)$$

where M_s is the string scale, so that having a low $m_{3/2} = O(10 \text{ TeV})$ imposes the circle to be extremely large, $R = O(10^{17})$ [6]. Such large directions yield towers of light Kaluza-Klein states and a problem arises from those charged under some gauge group factor G^i . In general, their contributions to the quantum corrections to the inverse squared gauge coupling is proportional to the very large volume and invalidates the use of perturbation theory.

To be specific, let us consider in heterotic string the 1-loop low energy running gauge coupling $g_i(\mu)$, which satisfies [7]

$$\frac{16\pi^2}{g_i^2(\mu)} = k^i \frac{16\pi^2}{g_s^2} + b^i \ln \frac{M_s^2}{\mu^2} + \Delta^i. \quad (2)$$

In this expression, g_s is the string coupling and k^i is the Kac-Moody level of G^i . The logarithmic contribution, which depends on the energy scale μ , arises from the massless states and is proportional to the β -function coefficient b^i , while the massive modes yield the threshold corrections Δ^i . The main contributions to the latter arise from the light Kaluza-Klein states, which for a single large radius yield

$$\Delta^i = C^i R - b^i \ln R^2 + O\left(\frac{1}{R}\right), \quad (3)$$

where $C^i = Cb^i - C'k^i$, for some non-negative C and C' that depend on other moduli. When $C^i = O(1)$, requiring in Eq. (2) the loop correction to be small compared to the tree level contribution imposes $g_s^2 R < 1$. In other words, for perturbation theory to be valid, the string coupling must be extremely weak, $g_s < O(10^{-6.5})$. If $C^i > 0$, which implies G^i is not asymptotically free, Eq. (2) imposes the running gauge coupling to be essentially free, $g_i(\mu) = O(g_s)$, and G^i describes a hidden gauge group. However, if $C^i < 0$, which is the case if G^i is asymptotically free, the very large tree level contribution proportional to $1/g_s^2$ must cancel $C^i R$, up to very high accuracy, for the running gauge coupling to be of order 1 and have a chance to describe realistic gauge interactions. This unnatural fine-tuning is a manifestation of the so-called “decompactification problem”, which actually arises generically, when a submanifold of the internal space is large, compared to the string scale, *i.e.* when the internal conformal field theory allows a geometrical interpretation in terms of a compactified space.

To avoid the above described behavior, C^i can be required to vanish. This is trivially the case in the $\mathcal{N} = 4$ supersymmetric theories, where actually $b^i = 0$ and $\Delta^i = 0$. The condition $C^i = 0$ remains valid in the theories realizing the $\mathcal{N} = 4 \rightarrow \mathcal{N} = 2$ spontaneous breaking, provided $\mathcal{N} = 4$ is recovered when the volume is sent to infinity [8]. In this case, the threshold corrections scale logarithmically with the volume and no fine-tuning is required for perturbation theory to be valid. In

Sect. 2, we review the construction of models that realize an $\mathcal{N} = 1 \rightarrow \mathcal{N} = 0$ spontaneous breaking at a low scale $m_{3/2}$, while avoiding the decompactification problem. The corresponding threshold corrections are computed in Sect. 3 [9, 10].

2 The non-supersymmetric $\mathbb{Z}_2 \times \mathbb{Z}_2$ models

In the present work, we focus on heterotic string backgrounds in four-dimensional Minkowski space and analyze the gauge coupling threshold corrections. At 1-loop, their formal expression is [7, 11, 12]

$$\Delta^i = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \left(\frac{1}{2} \sum_{a,b} \mathcal{Q}_{[b]}^{[a]}(2\nu) \left(\mathcal{P}_i^2(2\bar{w}) - \frac{k^i}{4\pi\tau_2} \right) \tau_2 Z_{[b]}^{[a]}(2\nu, 2\bar{w}) - b^i \right) \Big|_{\nu=\bar{w}=0} + b^i \log \frac{2e^{1-\gamma}}{\pi\sqrt{27}}, \quad (4)$$

where \mathcal{F} is the fundamental domain of $SL(2, \mathbb{Z})$ and $Z_{[b]}^{[a]}(2\nu, 2\bar{w})$ is a refined partition function for given spin structure $(a, b) \in \mathbb{Z}_2 \times \mathbb{Z}_2$. $\mathcal{P}_i^2(2\bar{w})$ acts on the right-moving sector as the squared charge operator of the gauge group factor G^i , while $\mathcal{Q}_{[b]}^{[a]}(2\nu)$ acts on the left-moving sector as the helicity operator,²

$$\mathcal{Q}_{[b]}^{[a]}(2\nu) = \frac{1}{16\pi^2} \frac{\partial_\nu^2 (\theta_{[b]}^{[a]}(2\nu))}{\theta_{[b]}^{[a]}(2\nu)} - \frac{i}{\pi} \partial_\tau \log \eta \equiv \frac{i}{\pi} \partial_\tau \left(\log \frac{\theta_{[b]}^{[a]}(2\nu)}{\eta} \right). \quad (5)$$

From now on, we consider $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold models [2] or fermionic constructions [3] in which the marginal deformations parameterized by the Kähler and complex structures $T_I, U_I, I = 1, 2, 3$, associated to the three internal 2-tori are switched on [9, 14]. In both cases, orbifolds or “moduli-deformed fermionic constructions”, $\mathcal{N} = 1$ supersymmetry is spontaneously broken by a stringy Scherk-Schwarz mechanism [4]. The associated genus-1 refined partition function is

$$Z(2\nu, 2\bar{w}) = \frac{1}{\tau_2(\eta\bar{\eta})^2} \times \quad (6)$$

$$\frac{1}{2} \sum_{a,b} \frac{1}{2} \sum_{H_1, G_1} \frac{1}{2} \sum_{H_2, G_2} (-1)^{a+b+ab} \frac{\theta_{[b]}^{[a]}(2\nu)}{\eta} \frac{\theta_{[b+G_1]}^{[a+H_1]}}{\eta} \frac{\theta_{[b+G_2]}^{[a+H_2]}}{\eta} \frac{\theta_{[b+G_3]}^{[a+H_3]}}{\eta} \times$$

$$\frac{1}{2^N} \sum_{h_I^i, g_I^i} S_L \left[\begin{smallmatrix} a, h_I^i, H_I \\ b, g_I^i, G_I \end{smallmatrix} \right] Z_{2,2} \left[\begin{smallmatrix} h_1^i & H_1 \\ g_1^i & G_1 \end{smallmatrix} \right] Z_{2,2} \left[\begin{smallmatrix} h_2^i & H_2 \\ g_2^i & G_2 \end{smallmatrix} \right] Z_{2,2} \left[\begin{smallmatrix} h_3^i & H_3 \\ g_3^i & G_3 \end{smallmatrix} \right] Z_{0,16} \left[\begin{smallmatrix} h_I^i, H_I \\ g_I^i, G_I \end{smallmatrix} \right] (2\bar{w}),$$

where our notations are as follows :

² Our conventions for the Jacobi functions $\theta_{[b]}^{[a]}(\nu|\tau)$ (or $\theta_\alpha(\nu|\tau)$, $\alpha = 1, \dots, 4$) and Dedekind function can be found in [13].

- The $Z_{2,2}$ conformal blocks arise from the three internal 2-tori. The genus-1 surface having two non-trivial cycles, $(h_I^i, g_I^i) \in \mathbb{Z}_2 \times \mathbb{Z}_2$, $i = 1, 2$, $I = 1, 2, 3$ denote associated shifts of the six coordinates. Similarly, $(H_I, G_I) \in \mathbb{Z}_2 \times \mathbb{Z}_2$ refer to the twists, where we have defined for convenience $(H_3, G_3) \equiv (-H_1 - H_2, -G_1 - G_2)$. Explicitly, we have

$$Z_{2,2} \left[\begin{smallmatrix} h_I^1, h_I^2 \\ g_I^1, g_I^2 \end{smallmatrix} \middle| \begin{smallmatrix} H_I \\ G_I \end{smallmatrix} \right] = \begin{cases} \frac{\Gamma_{2,2} \left[\begin{smallmatrix} h_I^1, h_I^2 \\ g_I^1, g_I^2 \end{smallmatrix} \right] (T_I, U_I)}{(\eta \bar{\eta})^2}, & \text{when } (H_I, G_I) = (0, 0) \bmod 2, \\ \frac{4\eta \bar{\eta}}{\theta \left[\begin{smallmatrix} 1-H_I \\ 1-G_I \end{smallmatrix} \right] \bar{\theta} \left[\begin{smallmatrix} 1-H_I \\ 1-G_I \end{smallmatrix} \right]} \delta \left[\begin{smallmatrix} h_I^1 & H_I \\ g_I^1 & G_I \end{smallmatrix} \right]_{0 \bmod 2} \delta \left[\begin{smallmatrix} h_I^2 & H_I \\ g_I^2 & G_I \end{smallmatrix} \right]_{0 \bmod 2} & \text{otherwise,} \end{cases} \quad (7)$$

where $\Gamma_{2,2}$ is a shifted lattice that depends on the Kähler and complex structure moduli T_I, U_I of the I^{th} 2-torus. The arguments of the Kronecker symbols are determinants.

- When defining each model, linear constraints on the shifts (h_I^i, g_I^i) and twists (H_I, G_I) may be imposed, leaving effectively N independent shifts.
- $Z_{0,16}$ denotes the contribution of the 32 extra right-moving world sheet fermions. Its dependance on the shifts and twists may generate discrete Wilson lines, which break partially $E_8 \times E_8$ or $SO(32)$.
- The first line contains the contribution of the spacetime light-cone bosons, while the second is that of the left-moving fermions.
- S_L is a conformal block-dependent sign that implements the stringy Scherk-Schwarz mechanism. A choice of S_L that correlates the spin structure (a, b) to some shift (h_I^i, g_I^i) implements the $\mathcal{N} = 1 \rightarrow \mathcal{N} = 0$ spontaneous breaking.

The $\mathbb{Z}_2 \times \mathbb{Z}_2$ models contain three $\mathcal{N} = 2$ sectors. For the decompactification problem not to arise, we impose one of them to be realized as a spontaneously broken phase of $\mathcal{N} = 4$. This can be done by demanding the \mathbb{Z}_2 action characterized by (H_2, G_2) to be free. The associated generator twists the 2nd and 3rd 2-tori (*i.e.* the directions X^6, X^7, X^8, X^9 in bosonic language) and shifts some direction(s) of the 1st 2-torus, say X^5 only. To simplify our discussion, we take the generator of the other \mathbb{Z}_2 , whose action is characterized by (H_1, G_1) , to not be free : It twists the 1st and 3rd 2-tori, and fixes the 2nd one. Similarly, we suppose that the product of the two generators, whose action is characterized by (H_3, G_3) , twists the 1st and 2nd 2-tori, and fixes the 3rd one. These restrictions impose the moduli T_2, U_2 and T_3, U_3 not to be far from 1, in order to avoid the decompactification problem to occur from the remaining two $\mathcal{N} = 2$ sectors. However, our care in choosing the orbifold action is allowing us to take the volume of the 1st 2-torus to be large.

The above remarks have an important consequence, since the final stringy Scherk-Schwarz mechanism responsible of the $\mathcal{N} = 1 \rightarrow \mathcal{N} = 0$ spontaneous breaking must involve the moduli T_1, U_1 only, for the gravitino mass to be light. Thus, this breaking must be implemented *via* a shift along the 1st 2-torus, say X^4 , and a non-trivial choice of S_L . Therefore, the sector $(H_1, G_1) = (0, 0)$ realizes the pattern of

spontaneous breaking $\mathcal{N} = 4 \rightarrow \mathcal{N} = 2 \rightarrow \mathcal{N} = 0$, while the other two $\mathcal{N} = 2$ sectors, which have 2nd and 3rd 2-tori respectively fixed, are independent of T_1 and U_1 and thus remain supersymmetric. As a result, we have in the two following independent modular orbits :

$$\begin{aligned} S_L &= (-1)^{ag_1^1 + bh_1^1 + h_1^1 g_1^1}, \text{ when } (H_1, G_1) = (0, 0), \\ S_L &= 1, \text{ when } (H_1, G_1) \neq (0, 0). \end{aligned} \quad (8)$$

Given the fact that we have imposed $(h_1^2, g_1^2) \equiv (H_2, G_2)$, the 1st 2-torus lattice takes the explicit form

$$\begin{aligned} \Gamma_{2,2} \left[\begin{smallmatrix} h_1^1, H_2 \\ g_1^1, G_2 \end{smallmatrix} \right] (T_1, U_1) &= \sum_{m^1, n^1} (-1)^{m^1 g_1^1 + m^2 G_2} e^{2i\pi \tau \left[m^1 \left(n^1 + \frac{1}{2} h_1^1 \right) + m^2 \left(n^2 + \frac{1}{2} H_2 \right) \right]} \times \\ &e^{-\frac{\pi \tau_2}{\text{Im } T_1 \text{Im } U_1} |T_1 \left(n^1 + \frac{1}{2} h_1^1 \right) + T_1 U_1 \left(n^2 + \frac{1}{2} H_2 \right) + U_1 m^1 - m^2|^2}. \end{aligned} \quad (9)$$

This expression can be used to find the squared scales of spontaneous $\mathcal{N} = 4 \rightarrow \mathcal{N} = 2$ and $\mathcal{N} = 2 \rightarrow \mathcal{N} = 0$ breaking. For $\text{Re}(U_1) \in (-\frac{1}{2}, \frac{1}{2}]$, they are

$$\frac{M_s^2}{\text{Im } T_1 \text{Im } U_1}, \quad m_{3/2}^2 = \frac{|U_1|^2 M_s^2}{\text{Im } T_1 \text{Im } U_1}, \quad (10)$$

where the latter is nothing but the gravitino mass squared of the full $\mathcal{N} = 0$ theory. For these scales to be small compared to M_s , we consider the regime $\text{Im } T_1 \gg 1$, $U_1 = O(i)$.

3 Threshold corrections

The threshold corrections can be evaluated in each conformal block [9]. Starting with those where $(H_1, G_1) = (0, 0)$, the discussion is facilitated by summing over the spin structures. Focussing on the relevant parts of the refined partition function Z , we have

$$\begin{aligned} \frac{1}{2} \sum_{a,b} (-1)^{a+b+ab} (-1)^{ag_1^1 + bh_1^1 + h_1^1 g_1^1} \theta \left[\begin{smallmatrix} a \\ b \end{smallmatrix} \right] (2v) \theta \left[\begin{smallmatrix} a \\ b \end{smallmatrix} \right] \theta \left[\begin{smallmatrix} a+H_2 \\ b+G_2 \end{smallmatrix} \right] \theta \left[\begin{smallmatrix} a-H_2 \\ b-G_2 \end{smallmatrix} \right] = \\ (-1)^{h_1^1 g_1^1 + G_2(1+h_1^1+H_2)} \theta \left[\begin{smallmatrix} 1-h_1^1 \\ 1-g_1^1 \end{smallmatrix} \right]^2 (v) \theta \left[\begin{smallmatrix} 1-h_1^1+H_2 \\ 1-g_1^1+G_2 \end{smallmatrix} \right]^2 (v), \end{aligned} \quad (11)$$

which shows how many odd $\theta_1(v) \equiv \theta \left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix} \right] (v)$ functions (or equivalently how many fermionic zero modes in the path integral) arise for given shift (h_1^1, g_1^1) and twist (H_2, G_2) .

Conformal block A : $(h_1^1, g_1^1) = (0, 0), (H_2, G_2) = (0, 0)$

This block is proportional to $\theta_{[1]}^4(v) = O(v^4)$. Up to an overall factor $1/2^3$, it is the contribution of the $\mathcal{N} = 4$ spectrum of the parent theory, when neither the $\mathbb{Z}_2 \times \mathbb{Z}_2$ action nor the stringy Scherk-Schwarz mechanism are implemented. Therefore, it does not contribute to the 1-loop gauge couplings.

Conformal blocks B : $(h_1^1, g_1^1) \neq (0, 0), (H_2, G_2) = (0, 0)$

They are proportional to $\theta_{[1-g_1^1]}^{[1-h_1^1]}(v) = O(1)$. The parity of the winding number along the compact direction X^4 being h_1^1 , the blocks with $h_1^1 = 1$ involve states, which are super massive compared to the pure Kaluza-Klein modes. These blocks are therefore exponentially suppressed, compared to the block $(h_1^1, g_1^1) = (0, 1)$. Up to an overall factor $1/2^2$, the latter arises from the spectrum considered in the conformal block A, but in the $\mathcal{N} = 4 \rightarrow \mathcal{N} = 0$ spontaneously broken phase, and contributes to the gauge couplings.

Conformal blocks C : $(h_1^1, g_1^1) = (0, 0), (H_2, G_2) \neq (0, 0)$

They are proportional to $\theta_{[1]}(v)^2 \theta_{[1-H_2]}^{[1-G_2]}(v) = O(v^2)$ and do contribute to Δ^i , due to the action of the helicity operator. Reasoning as in the previous case, the parity of the winding number along the compact direction X^5 is H_2 , which implies the blocks with $H_2 = 1$ yield exponentially suppressed contributions, compared to that associated to the block $(H_2, G_2) = (0, 1)$. Up to an overall factor $1/2^2$, the latter arises from a spectrum realizing the spontaneous $\mathcal{N} = 4 \rightarrow \mathcal{N}_C = 2$ breaking, which contributes to the couplings.

Conformal blocks D : $(h_1^1, g_1^1) = (H_2, G_2) \neq (0, 0)$

They are proportional to $\theta_{[1-G_2]}^{[1-H_2]}(v) \theta_{[1]}(v)^2 = O(v^2)$. The situation is identical to that of the conformal blocks C, except that the generator of the \mathbb{Z}_2 free action responsible of the partial spontaneous breaking of $\mathcal{N} = 4$ twists X^6, X^7, X^8, X^9 and shifts X^4, X^5 . The dominant contribution to the threshold corrections arises again from the block $(H_2, G_2) = (0, 1)$, which describes a spectrum realizing the spontaneous $\mathcal{N} = 4 \rightarrow \mathcal{N}_D = 2$ breaking.

Conformal blocks E : $\begin{vmatrix} h_1^1 & H_2 \\ g_1^1 & G_2 \end{vmatrix} \neq 0$

The remaining conformal blocks have non-trivial determinant $\begin{vmatrix} h_1^1 & H_2 \\ g_1^1 & G_2 \end{vmatrix}$, which implies $\theta_{[1-g_1^1]}^{[1-h_1^1]}(v) \theta_{[1-g_1^1+G_2]}^{[1-h_1^1+H_2]}(v) = O(1)$. However, this condition is also saying that $(h_1^1, H_2) \neq (0, 0)$, which means the modes in these blocks have non-trivial winding number(s) along X^4, X^5 or both. Therefore, their contributions to the gauge couplings are non-trivial but exponentially suppressed.

Having analyzed all conformal blocks satisfying $(H_1, G_1) = (0, 0)$, we proceed with the study of the modular orbit $(H_1, G_1) \neq (0, 0)$, where the sign S_L is trivial.

Since the 1st 2-torus is twisted, these blocks are independent of the moduli T_1, U_1 and thus $m_{3/2}$. They can be analyzed as in the case of $\mathbb{Z}_2 \times \mathbb{Z}_2$, $\mathcal{N} = 1$ supersymmetric models. Actually, summing over the spin structures, the relevant terms in the refined partition function Z become

$$\frac{1}{2} \sum_{a,b} (-1)^{a+b+ab} \theta \begin{bmatrix} a \\ b \end{bmatrix} (2v) \theta \begin{bmatrix} a+H_1 \\ b+G_1 \end{bmatrix} \theta \begin{bmatrix} a+H_2 \\ b+G_2 \end{bmatrix} \theta \begin{bmatrix} a-H_1-H_2 \\ b-G_1-G_2 \end{bmatrix} =$$

$$(-1)^{(G_1+G_2)(1+H_1+H_2)} \theta \begin{bmatrix} 1 \\ 1 \end{bmatrix} (v) \theta \begin{bmatrix} 1-H_1 \\ 1-G_1 \end{bmatrix} (v) \theta \begin{bmatrix} 1-H_2 \\ 1-G_2 \end{bmatrix} (v) \theta \begin{bmatrix} 1+H_1+H_2 \\ 1+G_1+G_2 \end{bmatrix} (v), \quad (12)$$

which invites us to split the discussion in three parts.

$\mathcal{N} = 2$ conformal blocks, with fixed 2nd 2-torus : $(H_2, G_2) = (0, 0)$

They are proportional to $\theta \begin{bmatrix} 1 \\ 1 \end{bmatrix}^2 (v) \theta \begin{bmatrix} 1-H_1 \\ 1-G_1 \end{bmatrix}^2 (v) = O(v^2)$. The 2nd internal 2-torus is fixed by the non-free action of the \mathbb{Z}_2 characterized by (H_1, G_1) . Adding the conformal block A , we obtain an $\mathcal{N} = 2$ sector of the theory, up to an overall factor $1/2$ associated to the second \mathbb{Z}_2 . This spectrum leads to non-trivial corrections to the gauge couplings.

$\mathcal{N} = 2$ conformal blocks, with fixed 3rd 2-torus : $(H_1, G_1) = (H_2, G_2)$

They are proportional to $\theta \begin{bmatrix} 1 \\ 1 \end{bmatrix}^2 (v) \theta \begin{bmatrix} 1-H_1 \\ 1-G_1 \end{bmatrix}^2 (v) = O(v^2)$. Actually, $(H_3, G_3) = (0, 0)$, which means that the 3rd 2-torus is fixed by the combined action of the generators of the two \mathbb{Z}_2 's. Adding the conformal block A , one obtains the last $\mathcal{N} = 2$ sector of the theory, up to an overall factor $1/2$. Again, this spectrum yields a non-trivial contribution to the gauge couplings.

$\mathcal{N} = 1$ conformal blocks : $\begin{vmatrix} H_1 & H_2 \\ G_1 & G_2 \end{vmatrix} \neq 0$

The remaining blocks have non-trivial determinant, $\begin{vmatrix} H_1 & H_2 \\ G_1 & G_2 \end{vmatrix} \neq 0$, which implies they are proportional to $\theta \begin{bmatrix} 1 \\ 1 \end{bmatrix} (v) \theta \begin{bmatrix} 1-H_1 \\ 1-G_1 \end{bmatrix} (v) \theta \begin{bmatrix} 1-H_2 \\ 1-G_2 \end{bmatrix} (v) \theta \begin{bmatrix} 1+H_1+H_2 \\ 1+G_1+G_2 \end{bmatrix} (v) = O(v)$. Acting on them with the helicity operator, the result is proportional to

$$\partial_v^2 \left(\theta \begin{bmatrix} 1 \\ 1 \end{bmatrix} (v) \theta \begin{bmatrix} 1-H_1 \\ 1-G_1 \end{bmatrix} (v) \theta \begin{bmatrix} 1-H_2 \\ 1-G_2 \end{bmatrix} (v) \theta \begin{bmatrix} 1+H_1+H_2 \\ 1+G_1+G_2 \end{bmatrix} (v) \right) \Big|_{v=0} \propto$$

$$\partial_v^2 \left(\theta_1(v) \theta_2(v) \theta_3(v) \theta_4(v) \right) \Big|_{v=0} = 0, \quad (13)$$

thanks to the oddness of $\theta_1(v)$ and evenness of $\theta_{2,3,4}(v)$. Thus, these conformal blocks do not contribute to the thresholds.

In the class of models we consider, the effective running gauge coupling associated to some gauge group factor G^i has a universal form at 1-loop [9]. It can be elegantly expressed in terms of three moduli-dependent squared mass scales arising from the corrections associated to the conformal blocks B, C, D ,

$$M_B^2 = \frac{M_s^2}{|\theta_2(U_1)|^4 \text{Im } T_1 \text{Im } U_1}, M_C^2 = \frac{M_s^2}{|\theta_4(U_1)|^4 \text{Im } T_1 \text{Im } U_1}, M_D^2 = \frac{M_s^2}{|\theta_3(U_1)|^4 \text{Im } T_1 \text{Im } U_1}, \quad (14)$$

which are of order $m_{3/2}^2$, and two more scales

$$M_I^2 = \frac{M_s^2}{16|\eta(T_I)|^4 |\eta(U_I)|^4 \text{Im } T_I \text{Im } U_I}, \quad I = 2, 3, \quad (15)$$

of order M_s^2 that encode the contributions of the $\mathcal{N} = 2$ sectors associated to the fixed 2nd and 3rd internal 2-tori. It is also useful to introduce a “renormalized string coupling” [11],

$$\frac{16\pi^2}{g_{\text{renor}}^2} = \frac{16\pi^2}{g_s^2} - \frac{1}{2}Y(T_2, U_2) - \frac{1}{2}Y(T_3, U_3),$$

where $Y(T, U) = \frac{1}{12} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \Gamma_{2,2}(T, U) \left[\left(\bar{E}_2 - \frac{3}{\pi\tau_2} \right) \frac{\bar{E}_4 \bar{E}_6}{\bar{\eta}^{24}} - \bar{j} + 1008 \right], \quad (16)$

in which $\Gamma_{2,2} = \Gamma_{2,2}^{[0,0]}$ is the unshifted lattice, while for $q = e^{2i\pi\tau}$, $E_{2,4,6} = 1 + O(q)$ are holomorphic Eisenstein series of modular weights 2, 4, 6 and $j = 1/q + 744 + O(q)$ is holomorphic and modular invariant. The inverse squared 1-loop gauge coupling at energy scale $Q^2 = \mu^2 \frac{\pi^2}{4}$ is then

$$\begin{aligned} \frac{16\pi^2}{g_i^2(Q)} = & k^i \frac{16\pi^2}{g_{\text{renor}}^2} - \frac{b_B^i}{4} \ln \left(\frac{Q^2}{Q^2 + M_B^2} \right) - \frac{b_C^i}{4} \ln \left(\frac{Q^2}{Q^2 + M_C^2} \right) - \frac{b_D^i}{4} \ln \left(\frac{Q^2}{Q^2 + M_D^2} \right) \\ & - \frac{b_2^i}{2} \ln \left(\frac{Q^2}{M_2^2} \right) - \frac{b_3^i}{2} \ln \left(\frac{Q^2}{M_3^2} \right) + O \left(\frac{m_{3/2}^2}{M_s^2} \right), \end{aligned} \quad (17)$$

which depends only on five model-dependent β -function coefficients and the Kac-Moody level. In this final result, we have shifted $M_{B,C,D}^2 \rightarrow Q^2 + M_{B,C,D}^2$ in order to implement the thresholds at which the sectors B , C or D decouple, *i.e.* when Q exceeds M_B , M_C or M_D . Thus, this expression is valid as long as Q is lower than the mass of the heavy states we have neglected the exponentially suppressed contributions *i.e.* the string or GUT scale, depending on the model. Taking Q lower than at least one of the scales M_B , M_C or M_D , the r.h.s. of Eq. (17) scales as $\ln \text{Im } T_1$, which is the logarithm of the large 1st 2-torus volume, as expected for the decompactification problem not to arise.

To conclude, we would like to mention two important remarks. First of all, we stress that the $\mathbb{Z}_2 \times \mathbb{Z}_2$ models, where a \mathbb{Z}_2 is freely acting and a stringy Scherk-Schwarz mechanism responsible of the final breaking of $\mathcal{N} = 1$ takes place, have non-chiral massless spectra. This is due to the fact that in the $\mathcal{N} = 1$, $\mathbb{Z}_2 \times \mathbb{Z}_2$ models, chiral families occur from twisted states localized at fixed points. In the models we have considered, fixed points localized on the 2nd and 3rd 2-tori can arise but are independent of the moduli T_1, U_1 *i.e.* $m_{3/2}$. Thus, taking the large volume

limit of the 1st 2-torus, where $\mathcal{N} = 2$ supersymmetry is recovered, one concludes that the twisted states are actually hypermultiplets *i.e.* couples of families and anti-families.

Second, we point out that in the models analyzed in the present work, the conformal block B is the only non-supersymmetric and non-negligible contribution to the partition function Z , and thus to the 1-loop effective potential. In Ref. [10, 15], it is shown that in some models, the latter is positive semi-definite. The motion of the moduli T_2, U_2 and T_3, U_3 is thus attracted to points [?], where the effective potential vanishes, allowing $m_{3/2}$ to be arbitrary. In other words, the defining properties of the no-scale models, namely arbitrariness of the supersymmetry breaking scale $m_{3/2}$ in flat space, which are valid at tree level, are extended to the 1-loop level. This very fact, characteristic of the so-called “super no-scale models”, may have interesting consequences on the smallness of a cosmological constant generated at higher orders. In Ref. [17], other models having 1-loop vanishing cosmological constant are also considered, which however suffer from the decompactification problem.

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